

# Exponents and Powers

CHAPTER

# 12

## 12.1 Introduction

### Do you know?

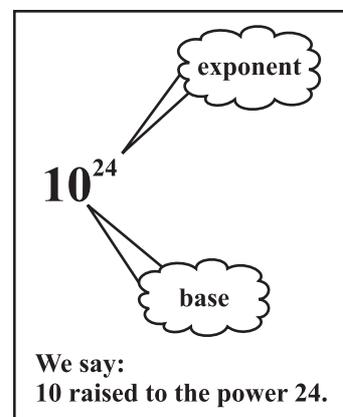
Mass of earth is 5,970,000,000,000, 000, 000, 000, 000 kg. We have already learnt in earlier class how to write such large numbers more conveniently using exponents, as,  $5.97 \times 10^{24}$  kg.

We read  $10^{24}$  as 10 raised to the power 24.

We know  $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

and  $2^m = 2 \times 2 \times 2 \times 2 \times \dots \times 2 \times 2 \dots$  ( $m$  times)

Let us now find what is  $2^{-2}$  is equal to?



## 12.2 Powers with Negative Exponents

You know that,

$$10^2 = 10 \times 10 = 100$$

$$10^1 = 10 = \frac{100}{10}$$

$$10^0 = 1 = \frac{10}{10}$$

$$10^{-1} = ?$$

Continuing the above pattern we get,  $10^{-1} = \frac{1}{10}$

Similarly

$$10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

What is  $10^{-10}$  equal to?

Exponent is a negative integer.

As the exponent decreases by 1, the value becomes one-tenth of the previous value.

Now consider the following.



$$3^3 = 3 \times 3 \times 3 = 27$$

$$3^2 = 3 \times 3 = 9 = \frac{27}{3}$$

$$3^1 = 3 = \frac{9}{3}$$

$$3^0 = 1 = \frac{3}{3}$$

The previous number is divided by the base 3.

So looking at the above pattern, we say

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

You can now find the value of  $2^{-2}$  in a similar manner.

We have,

$$10^{-2} = \frac{1}{10^2} \quad \text{or} \quad 10^2 = \frac{1}{10^{-2}}$$

$$10^{-3} = \frac{1}{10^3} \quad \text{or} \quad 10^3 = \frac{1}{10^{-3}}$$

$$3^{-2} = \frac{1}{3^2} \quad \text{or} \quad 3^2 = \frac{1}{3^{-2}} \quad \text{etc.}$$

In general, we can say that for any non-zero integer  $a$ ,  $a^{-m} = \frac{1}{a^m}$ , where  $m$  is a positive integer.  $a^{-m}$  is the multiplicative inverse of  $a^m$ .



### TRY THESE

Find the multiplicative inverse of the following.

- (i)  $2^{-4}$       (ii)  $10^{-5}$       (iii)  $7^{-2}$       (iv)  $5^{-3}$       (v)  $10^{-100}$

We learnt how to write numbers like 1425 in expanded form using exponents as  $1 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$ .

Let us see how to express 1425.36 in expanded form in a similar way.

$$\begin{aligned} \text{We have } 1425.36 &= 1 \times 1000 + 4 \times 100 + 2 \times 10 + 5 \times 1 + \frac{3}{10} + \frac{6}{100} \\ &= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 5 \times 1 + 3 \times 10^{-1} + 6 \times 10^{-2} \end{aligned}$$

$$10^{-1} = \frac{1}{10}, \quad 10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

### TRY THESE

Expand the following numbers using exponents.

- (i) 1025.63      (ii) 1256.249

### 12.3 Laws of Exponents

We have learnt that for any non-zero integer  $a$ ,  $a^m \times a^n = a^{m+n}$ , where  $m$  and  $n$  are natural numbers. Does this law also hold if the exponents are negative? Let us explore.

(i) We know that  $2^{-3} = \frac{1}{2^3}$  and  $2^{-2} = \frac{1}{2^2}$

$a^{-m} = \frac{1}{a^m}$  for any non-zero integer  $a$ .

Therefore,  $2^{-3} \times 2^{-2} = \frac{1}{2^3} \times \frac{1}{2^2} = \frac{1}{2^3 \times 2^2} = \frac{1}{2^{3+2}} = 2^{-5}$

(ii) Take  $(-3)^{-4} \times (-3)^{-3}$

-5 is the sum of two exponents -3 and -2

$$(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3)^4 \times (-3)^3} = \frac{1}{(-3)^{4+3}} = (-3)^{-7}$$

$(-4) + (-3) = -7$

(iii) Now consider  $5^{-2} \times 5^4$

$$5^{-2} \times 5^4 = \frac{1}{5^2} \times 5^4 = \frac{5^4}{5^2} = 5^{4-2} = 5^{(2)}$$

$(-2) + 4 = 2$

In Class VII, you have learnt that for any non-zero integer  $a$ ,  $\frac{a^m}{a^n} = a^{m-n}$ , where  $m$  and  $n$  are natural numbers and  $m > n$ .

(iv) Now consider  $(-5)^{-4} \times (-5)^2$

$$(-5)^{-4} \times (-5)^2 = \frac{1}{(-5)^4} \times (-5)^2 = \frac{(-5)^2}{(-5)^4} = \frac{1}{(-5)^4 \times (-5)^{-2}}$$

$$= \frac{1}{(-5)^{4-2}} = (-5)^{-2}$$

$(-4) + 2 = -2$

In general, we can say that for any non-zero integer  $a$ ,  $a^m \times a^n = a^{m+n}$ , where  $m$  and  $n$  are integers.

#### TRY THESE

Simplify and write in exponential form.

(i)  $(-2)^{-3} \times (-2)^{-4}$     (ii)  $p^3 \times p^{-10}$     (iii)  $3^2 \times 3^{-5} \times 3^6$



On the same lines you can verify the following laws of exponents, where  $a$  and  $b$  are non zero integers and  $m, n$  are any integers.

(i)  $\frac{a^m}{a^n} = a^{m-n}$     (ii)  $(a^m)^n = a^{mn}$     (iii)  $a^m \times b^m = (ab)^m$

(iv)  $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$     (v)  $a^0 = 1$

These laws you have studied in Class VII for positive exponents only.

Let us solve some examples using the above Laws of Exponents.

**Example 1:** Find the value of

(i)  $2^{-3}$                       (ii)  $\frac{1}{3^{-2}}$

**Solution:**

(i)  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$       (ii)  $\frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

**Example 2:** Simplify

(i)  $(-4)^5 \times (-4)^{-10}$       (ii)  $2^5 \div 2^{-6}$

**Solution:**

(i)  $(-4)^5 \times (-4)^{-10} = (-4)^{(5-10)} = (-4)^{-5} = \frac{1}{(-4)^5}$       ( $a^m \times a^n = a^{m+n}$ ,  $a^{-m} = \frac{1}{a^m}$ )  
 (ii)  $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11}$       ( $a^m \div a^n = a^{m-n}$ )

**Example 3:** Express  $4^{-3}$  as a power with the base 2.

**Solution:** We have,  $4 = 2 \times 2 = 2^2$

Therefore,  $(4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6}$       [ $(a^m)^n = a^{mn}$ ]

**Example 4:** Simplify and write the answer in the exponential form.

(i)  $(2^5 \div 2^8)^5 \times 2^{-5}$                       (ii)  $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii)  $\frac{1}{8} \times (3)^{-3}$                       (iv)  $(-3)^4 \times \left(\frac{5}{3}\right)^4$

**Solution:**

(i)  $(2^5 \div 2^8)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5} = (2^{-3})^5 \times 2^{-5} = 2^{-15-5} = 2^{-20} = \frac{1}{2^{20}}$

(ii)  $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = [100]^{-3} = \frac{1}{100^3}$

[using the law  $a^m \times b^m = (ab)^m$ ,  $a^{-m} = \frac{1}{a^m}$ ]

(iii)  $\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \times (3)^{-3} = 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3}$

(iv)  $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4} = (-1)^4 \times 3^4 \times \frac{5^4}{3^4}$   
 $= (-1)^4 \times 5^4 = 5^4$       [ $(-1)^4 = 1$ ]

**Example 5:** Find  $m$  so that  $(-3)^{m+1} \times (-3)^5 = (-3)^7$

**Solution:**  $(-3)^{m+1} \times (-3)^5 = (-3)^7$   
 $(-3)^{m+1+5} = (-3)^7$   
 $(-3)^{m+6} = (-3)^7$

On both the sides powers have the same base different from 1 and  $-1$ , so their exponents must be equal.



Therefore,  $m + 6 = 7$   
 or  $m = 7 - 6 = 1$

**Example 6:** Find the value of  $\left(\frac{2}{3}\right)^{-2}$ .

**Solution:**  $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \frac{9}{4}$

**Example 7:** Simplify (i)  $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii)  $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

$a^n = 1$  only if  $n = 0$ . This will work for any  $a$  except  $a = 1$  or  $a = -1$ . For  $a = 1$ ,  $1^1 = 1^2 = 1^3 = 1^{-2} = \dots = 1$  or  $(1)^n = 1$  for infinitely many  $n$ . For  $a = -1$ ,  $(-1)^0 = (-1)^2 = (-1)^2 = (-1)^{-2} = \dots = 1$  or  $(-1)^p = 1$  for any even integer  $p$ .

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$$

In general,  $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

**Solution:**

(i)  $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = \{9 - 8\} \div 16 = \frac{1}{16}$$

(ii)  $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}} \times \frac{8^{-5}}{8^{-7}} = 5^{(-7)-(-5)} \times 8^{(-5)-(-7)}$

$$= 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$$

## EXERCISE 12.1

1. Evaluate.

(i)  $3^{-2}$

(ii)  $(-4)^{-2}$

(iii)  $\left(\frac{1}{2}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

(i)  $(-4)^5 \div (-4)^8$  (ii)  $\left(\frac{1}{2^3}\right)^2$

(iii)  $(-3)^4 \times \left(\frac{5}{3}\right)^4$  (iv)  $(3^{-7} \div 3^{-10}) \times 3^{-5}$  (v)  $2^{-3} \times (-7)^{-3}$

3. Find the value of.

(i)  $(3^0 + 4^{-1}) \times 2^2$  (ii)  $(2^{-1} \times 4^{-1}) \div 2^{-2}$  (iii)  $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$



$$(iv) (3^{-1} + 4^{-1} + 5^{-1})^0 \qquad (v) \left\{ \left( \frac{-2}{3} \right)^{-2} \right\}^2$$

4. Evaluate (i)  $\frac{8^{-1} \times 5^3}{2^{-4}}$  (ii)  $(5^{-1} \times 2^{-1}) \times 6^{-1}$

5. Find the value of  $m$  for which  $5^m \div 5^{-3} = 5^5$ .

6. Evaluate (i)  $\left\{ \left( \frac{1}{3} \right)^{-1} - \left( \frac{1}{4} \right)^{-1} \right\}^{-1}$  (ii)  $\left( \frac{5}{8} \right)^{-7} \times \left( \frac{8}{5} \right)^{-4}$

7. Simplify.

(i)  $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$  (ii)  $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

## 12.4 Use of Exponents to Express Small Numbers in Standard Form

Observe the following facts.

- The distance from the Earth to the Sun is 149,600,000,000 m.
- The speed of light is 300,000,000 m/sec.
- Thickness of Class VII Mathematics book is 20 mm.
- The average diameter of a Red Blood Cell is 0.000007 mm.
- The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
- The distance of moon from the Earth is 384, 467, 000 m (approx).
- The size of a plant cell is 0.00001275 m.
- Average radius of the Sun is 695000 km.
- Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
- Thickness of a piece of paper is 0.0016 cm.
- Diameter of a wire on a computer chip is 0.000003 m.
- The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m.

Identify very large and very small numbers from the above facts and write them in the adjacent table:

We have learnt how to express very large numbers in standard form in the previous class.

For example:  $150,000,000,000 = 1.5 \times 10^{11}$   
Now, let us try to express 0.000007 m in standard form.

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
-----	-----
-----	-----
-----	-----
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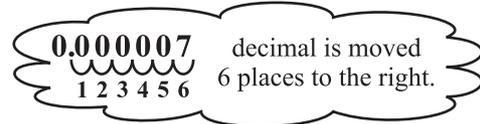
$$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

$$0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$$

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

$$\begin{aligned} 0.0016 &= \frac{16}{10000} = \frac{1.6 \times 10}{10^4} = 1.6 \times 10 \times 10^{-4} \\ &= 1.6 \times 10^{-3} \end{aligned}$$

Therefore, we can say thickness of paper is  $1.6 \times 10^{-3}$  cm.



**Again notice**

**0.0016** decimal is moved 3 places to the right.

### TRY THESE

1. Write the following numbers in standard form.

- (i) 0.000000564    (ii) 0.0000021    (iii) 21600000    (iv) 15240000

2. Write all the facts given in the standard form.

#### 12.4.1 Comparing very large and very small numbers

The diameter of the Sun is  $1.4 \times 10^9$  m and the diameter of the Earth is  $1.2756 \times 10^7$  m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

$$\text{Diameter of the Sun} = 1.4 \times 10^9 \text{ m}$$

$$\text{Diameter of the earth} = 1.2756 \times 10^7 \text{ m}$$

$$\text{Therefore } \frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 100}{1.2756} \text{ which is approximately } 100$$

So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red Blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

$$\text{Size of Red Blood cell} = 0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$$

$$\text{Size of plant cell} = 0.00001275 = 1.275 \times 10^{-5} \text{ m}$$

$$\text{Therefore, } \frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{-6-(-5)}}{1.275} = \frac{7 \times 10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} = \frac{1}{2} \text{ (approx.)}$$

So a red blood cell is half of plant cell in size.

Mass of earth is  $5.97 \times 10^{24}$  kg and mass of moon is  $7.35 \times 10^{22}$  kg. What is the total mass?

$$\begin{aligned} \text{Total mass} &= 5.97 \times 10^{24} \text{ kg} + 7.35 \times 10^{22} \text{ kg.} \\ &= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22} \\ &= 597 \times 10^{22} + 7.35 \times 10^{22} \\ &= (597 + 7.35) \times 10^{22} \\ &= 604.35 \times 10^{22} \text{ kg.} \end{aligned}$$

When we have to add numbers in standard form, we convert them into numbers with the same exponents.

The distance between Sun and Earth is  $1.496 \times 10^{11}$  m and the distance between Earth and Moon is  $3.84 \times 10^8$  m.

During solar eclipse moon comes in between Earth and Sun.

At that time what is the distance between Moon and Sun.

$$\begin{aligned}\text{Distance between Sun and Earth} &= 1.496 \times 10^{11} \text{ m} \\ \text{Distance between Earth and Moon} &= 3.84 \times 10^8 \text{ m} \\ \text{Distance between Sun and Moon} &= 1.496 \times 10^{11} - 3.84 \times 10^8 \\ &= 1.496 \times 1000 \times 10^8 - 3.84 \times 10^8 \\ &= (1496 - 3.84) \times 10^8 \text{ m} = 1492.16 \times 10^8 \text{ m}\end{aligned}$$

**Example 8:** Express the following numbers in standard form.

- (i) 0.000035      (ii) 4050000

**Solution:** (i)  $0.000035 = 3.5 \times 10^{-5}$       (ii)  $4050000 = 4.05 \times 10^6$

**Example 9:** Express the following numbers in usual form.

- (i)  $3.52 \times 10^5$       (ii)  $7.54 \times 10^{-4}$       (iii)  $3 \times 10^{-5}$

**Solution:**

- (i)  $3.52 \times 10^5 = 3.52 \times 100000 = 352000$   
 (ii)  $7.54 \times 10^{-4} = \frac{7.54}{10^4} = \frac{7.54}{10000} = 0.000754$   
 (iii)  $3 \times 10^{-5} = \frac{3}{10^5} = \frac{3}{100000} = 0.00003$

Again we need to convert numbers in standard form into a numbers with the same exponents.

## EXERCISE 12.2



- Express the following numbers in standard form.
  - 0.00000000000085
  - 0.000000000000942
  - 6020000000000000
  - 0.00000000837
  - 31860000000
- Express the following numbers in usual form.
  - $3.02 \times 10^{-6}$
  - $4.5 \times 10^4$
  - $3 \times 10^{-8}$
  - $1.0001 \times 10^9$
  - $5.8 \times 10^{12}$
  - $3.61492 \times 10^6$
- Express the number appearing in the following statements in standard form.
  - 1 micron is equal to  $\frac{1}{1000000}$  m.
  - Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
  - Size of a bacteria is 0.0000005 m
  - Size of a plant cell is 0.00001275 m
  - Thickness of a thick paper is 0.07 mm
- In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

## WHAT HAVE WE DISCUSSED?

1. Numbers with negative exponents obey the following laws of exponents.

(a)  $a^m \times a^n = a^{m+n}$       (b)  $a^m \div a^n = a^{m-n}$       (c)  $(a^m)^n = a^{mn}$

(d)  $a^m \times b^m = (ab)^m$       (e)  $a^0 = 1$       (f)  $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

2. Very small numbers can be expressed in standard form using negative exponents.